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SIMPLE HARMONIC SOURCE IN A SLIGHTLY IRREGULAR WAVEGUIDE,(U)

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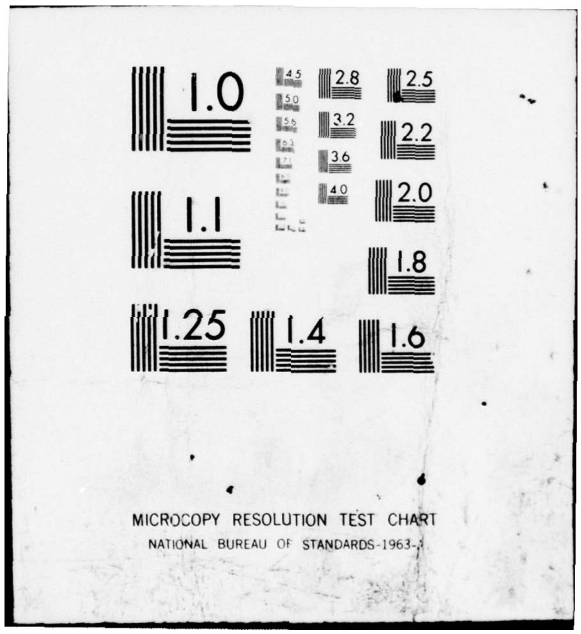
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SIMPLE HARMONIC SOURCE IN A SLIGHTLY IRREGULAR WAVEGUIDE

by

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C. S. Clay

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ABSTRACT

The normal mode solution of the problem of the radiation field of a simple harmonic source in a layered waveguide is extended to the case of small irregularities in the surface. The theory is used to calculate the radiation field of a 147.8 cps simple harmonic source in about 20 m of shallow water over a thick layer of unconsolidated sediment. The calculations indicate that the effects of small surface roughness, i.e., rms wave height about $\frac{1}{2}$ m (HALF M.), are to increase the attenuation as a function of range and to decrease the mode interference maxima and minima. The experimental dependence of the acoustical pressure upon source distance is similar to that calculated with the theory.

INTRODUCTION

Acoustical theory has been developed for laterally homogeneous waveguides. Real waveguides usually have irregularities of the boundaries and fluctuations of acoustical velocity within the layers. Since we desire to compare theory with experiment, the effects of inhomogeneities must be included in the theory. Eby <u>et al.</u> have shown that inhomogeneities cause fluctuations and increased attenuation for propagation in the waveguide.[1]

A typical shallow water waveguide experiment is shown in Fig. 1. The source can be driven by either a continuous wave or band limited noise. The acoustical signal is observed as a function of range. The signal is processed as the average of the signal squared and measured with a meter. If one has a fortunate combination of frequency, layer thickness, etc., then two modes are propagating in the waveguide, and the acoustical field due to a continuous wave source would give data as a function of range similar to the upper curve in Fig. 1 (marked cw). Repeated range runs with a band limited noise source give data similar to the lower curve. The decrease of the mode interference effects of $\langle e^2 \rangle$ with range for the noise driven source is due to loss of coherence of one mode relative to the other as a function of range. The comparison of theory and experiment for the continuous wave source and noise source in shallow water has been discussed by Tolstoy,[2]

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[1] R. K. Eby, A. O. Williams, Jr., R. P. Ryan, and P. Tamarkin, "Study of acoustic propagation in a two-layered model," J. Acoust. Soc. Am. <u>32</u>, 88-99 (1960).

[2] I. Tolstoy, "Shallow water test of the theory of layered wave guides," J. Acoust. Soc. Am. <u>30</u>, 348-361 (1958).

and Clay[3] has described the use of a band limited noise source to study a shallow water waveguide.

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[3] C. S. Clay, "Propagation of band-limited noise in a layered wave guide," J. Acoust. Soc. Am. 31, 1473–1479 (1959).

THEORY

The form of the solution of the acoustical field in a waveguide can be expressed with normal modes. The acoustical field ψ is expressed as a sum of the functions Z_m, R_m as follows:

$$\psi \left(r, z, t \right) = \sum_m Z_m \left(k_{mz} z \right) R_m \left(k_{mr} r \right) e^{i\omega t} \quad , \tag{1}$$

where

- k is ω/c,
- k_{mz} is the vertical component of k for the mth mode,
- k_{mr} is the horizontal component of k for the mth mode,
- r and z are cylindrical coordinates,
- ω is angular frequency in rad/sec, and
- c is velocity of sound in 1st layer.

The R_m's are radial functions that express the cylindrical spreading of the mth mode. The Z_m functions include the source receiver depth and the boundary conditions of the layered waveguide. Application of the boundary conditions yields the following phase integral:

$$\int_h k_{mz} \, dz = m\pi \quad . \tag{2}$$

For a uniform layer of thickness h and reflection phase angle χ_m at the interface, the phase integral is

$$k_{mz} h + \chi_m = m\pi \quad . \tag{3}$$

The vertical and horizontal components of k for $m = 1$ and $m = 2$ are compared in Fig. 2.

The acoustical pressure due to a continuous wave source is observed with a transducer. The transducer output is amplified, squared, averaged, and recorded, as shown in Fig. 3. Duplication of this process follows: The acoustical field at the receiver ψ is the sum of modes

$$\psi(r, z, t) = \sum_m \psi_m \quad . \tag{4}$$

ψ is squared and averaged over time to yield

$$\langle \psi^2 \rangle_t = \sum_m \langle \psi_m^2 \rangle_t + \sum_{m \neq n} \langle \psi_m \psi_n \rangle_t \quad . \tag{5}$$

The radial function $R(k_{mr} r)$ can be approximated at large range as

$$R_m(k_{mr} r) \simeq \frac{1}{\sqrt{r}} \, e^{\,i\left(k_m r + \frac{\pi}{4}\right)} \quad . \tag{6}$$

Since Z_m is independent of r and can be replaced by constants we have as an example for the 1st and 2nd modes the following:

$$\langle \psi^2 \rangle_t \simeq \frac{1}{r}\left[A_1^2 + A_2^2 + 2A_1 A_2 \cos\left(k_{1r} - k_{2r}\right) r\right] \tag{7}$$

An example $\langle\psi^2\rangle_t$ with A_1 about equal to A_2 is shown on the bottom of Fig. 3.[4] It is apparent from this figure and Eq. (3) that change of frequency or k causes changes in k_{1r} and k_{2r} that would not be expected to be equal. Thus, change of frequency causes changes of the distances between mode interference maxima and minima.

Let us now examine the effect of a sinusoidal surface on waveguide propagation by first considering a single surface reflection. In Fig. 4 we assume an incident wave with amplitude A_m and wave number k_m. The reflected wave in the specular direction has amplitude A'_m and wave number k_m. For small irregularity height, A'_m of a single reflected wave is approximately the following:[5-7]

$$A'_m \simeq A_m J_o \; 2 k_{mz} \zeta \quad , \tag{8}$$

where

$$\zeta(x) = \zeta \sin \kappa x \tag{9}$$

and

κ = wave number of the surface corrugation.

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[4] $\langle\ \rangle_t$ means average over t.

[5] Lord Rayleigh, Theory of Sound (Dover Publications, New York, N. Y., 1945), Vol. II, p. 89-96.

[6] C. Eckart, "The scattering of sound from the sea surface," J. Acoust. Soc. Am. 25, 566-570 (1953).

[7] E. O. LaCasce, Jr., and P. Tamarkin, "Underwater sound reflection from a corrugated surface," J. Appl. Phys. 27, 138-148 (1956).

The waves that are scattered in directions that do not satisfy the phase integral, Eq. (3), are not trapped and highly attenuated. Waves can be scattered from one mode to another. However, since the attenuation varies for different modes, one expects the modes with least attenuation to have the most energy at large range.

Assuming that A'_m/A_m is nearly one, the reflection coefficient is

$$\left| A'_m/A_m \right| = 1 - \delta_m \quad , \tag{10}$$

and δ_m is the decrement in amplitude for a single reflection. In water layer of thickness h, the distance between reflections is $2h \tan \Theta_m$. Thus, the change in amplitude ΔA in distance Δr is

$$\frac{\Delta A_m}{\Delta r} = - \frac{\delta_m}{2h \tan \Theta_m} A_m \quad , \tag{11}$$

where

$$\Theta_m = \arccos \left(k_{mz}/k \right) \quad . \tag{12}$$

Passing to the limit and integrating yields

$$A'_m = A_m e^{-\gamma_m r} \quad , \tag{13}$$

where

$$\gamma_m = \delta_m / \left(2h \tan \Theta_m \right) . \tag{14}$$

With Eqs. 8, 10, and 14

$$\gamma_m \simeq \frac{k_{mz}}{2k_{mr}h}\left[1 - \left|J_o\left(2k_{mz}\zeta\right)\right|\right] ; \qquad (15)$$

$$\gamma_m \simeq \frac{k_{mz}}{2k_{mr}h}\left(k_{mz}\zeta\right)^2 \quad \text{for} \quad k_{mz}\zeta \ll 1 \quad . \qquad (16)$$

The attenuation in a slightly irregular waveguide is dependent upon the wave height, components of the wave number, and layer thickness. The values of γ_m and A'_m are given for the sinusoidal surface. Actually the wave surface is more noiselike although the frequency spectrum of the surface is very narrow.[8,9] Following Saenger, average A'_m can be determined by integration over the distribution of ζ.[10] In this notation for a Gaussian sea surface, ζ has a Rayleigh distribution. One effect of the surface irregularities on a single mode is increase of the attenuation of propagation in that mode.

Irregularities also cause incoherence of the modes relative to each other. Recalling the phase integral expression, Eq. (3), let us assume that the thickness of the waveguide is perturbed by ζ so that

$$k_{mz}\left(h_o + \zeta\right) + \chi_m = m\pi \quad . \qquad (17)$$

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[8] G. Neumann, "On ocean wave spectra and a new method of forecasting wind-generated sea," U. S. Army, Beach Erosion Board Technical Memorandum No. 43 (December 1953), p. 42.

[9] H. W. Marsh, M. Schulkin, and S. G. Kneale, " Scattering of underwater sound by the sea surface, " J. Acoust. Soc. Am. 33, 334-340 (1961).

[10] R. A. Saenger, "Statistical properties of $\int p^2 dt$ for pulsed cw reflected from a rough sea surface," (A), J. Acoust. Soc. Am. 34, 1980 (1962).

For constant frequency ω and wave number k, changes of ζ require change of k_{mz} for satisfaction of Eq. (17). There are corresponding changes in the value of k_{mr}. In this we have assumed that the surface wavelengths of the irregularities are very long. Scrimger has demonstrated that the waveguide propagation is most sensitive to irregularities near the source and receiver.[11] Thus, we assume that we have water of depth $(h_o + \zeta)$ near the source and receiver. The effect of ζ on k_{mz} and k_{mr} for the 1st and 2nd modes is illustrated in Fig. 5. The mode interference terms $\langle\psi_m \psi_n\rangle_t$ in Eq. (5) have a dependence upon k_{mr} and range r like the following

$$\langle\psi_m \psi_n\rangle_t \sim \cos\left(k_{mr} - k_{nr}\right) r \quad . \tag{18}$$

The average of $\langle\psi_m \psi_n\rangle_t$ over a Gaussian distribution $D(\zeta)$ with

$$D \equiv \frac{1}{\sigma\sqrt{\pi}}\, e^{-\frac{\zeta^2}{\sigma^2}} \tag{19}$$

yields the following:

$$\langle\psi_m \psi_n\rangle_{t,\zeta} \sim e^{-\frac{\left(k_{mr} - k_{nr}\right)^2 r^2 \sigma^2}{4h^2}} \cos\left(k_{mr} - k_{nr}\right) r \tag{20}$$

for $\frac{\sigma}{h} \ll 1$.

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[11] J. A. Scrimger, "Signal amplitude and phase fluctuations induced by surface waves in ducted sound propagation," J. Acoust. Soc. Am. 33, 239-247 (1961).

Using (20) and (13) in Eq. (7) for the 1st and 2nd modes gives the following approximate expression for $\langle \psi^2 \rangle_{t,\zeta}$:

$$\langle \psi^2 \rangle_{t,\zeta} \simeq \frac{1}{\sqrt{r}} \left[A_1'^2 + A_2'^2 + 2A_1' + A_2' \, e^{-\frac{\left(k_{1r} - k_{2r}\right)^2 r^2 \sigma^2}{4h^2}} \cdot \cos\left(k_{1r} - k_{2r}\right) r \right] . \quad (21)$$

This equation is illustrated in Fig. 6. The curves compare $\langle \psi^2 \rangle$ for smooth and rough surfaces.

COMPARISON WITH EXPERIMENT

Experimental data, solid lines, are compared with theory, dotted lines, in Fig. 7. These data were taken in 22.6 m of water near Fire Island. The experimental gear was the same for both runs. (The source was not calibrated, and the levels were different for the two runs.) The sea was very calm for the data marked $\sigma \approx 0$. The sea state was about 2 with a good swell when the data on the lower graph were taken.

CONCLUSIONS

This procedure of estimating the effect of lateral inhomogeneities on the coherence of acoustical propagation in the waveguide is limited to extensive, slowly varying inhomogeneities.

Irregularities or lateral inhomogeneities in a waveguide produce two effects. First, the waveguide attenuation is increased. Second, the modes become incoherent relative to origin time and position. These effects are mode dependent.

The theoretical acoustical pressure curves as a function of range were compared with experimental data. The theoretical mode interferences decreased with range about the same amount as in experimental data for each of the surface wave conditions.

ACKNOWLEDGMENTS

Mrs. Irene Fiore and Mrs. Antoinette Dolfinger prepared the curves. I had many helpful conversations with Dr. A. Saenger and Dr. I. Tolstoy. In addition, Dr. Tolstoy furnished waveguide calculations for the Fire Island area.

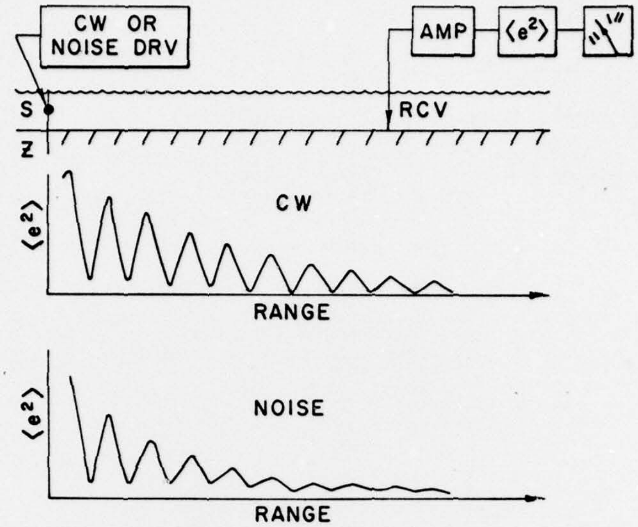

Figure 1. Shallow water waveguide experiment. The average signal squared $\langle e^2 \rangle$ a function of range for a continuous wave source (cw) and band limited noise source (noise).

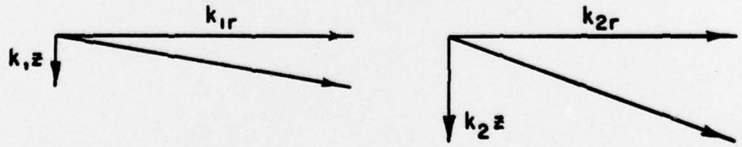

Figure 2. Vertical and horizontal components of k for $m = 1$ and $m = 2$.

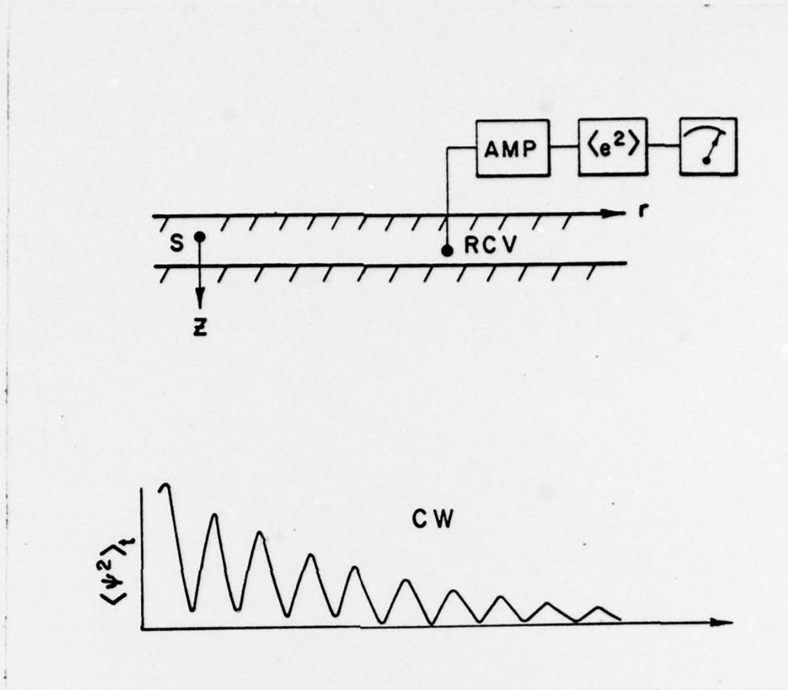

Figure 3. Average acoustical field as function of range. The two modes have nearly equal excitation.

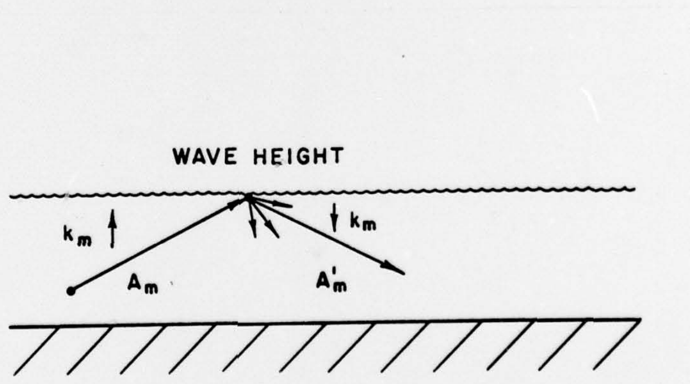

Figure 4. Scattering by irregular surface. The incident wave A_m, $k_m \uparrow$ is scattered into A'_m, $k_m \downarrow$ in the specular direction and other contributions in other directions.

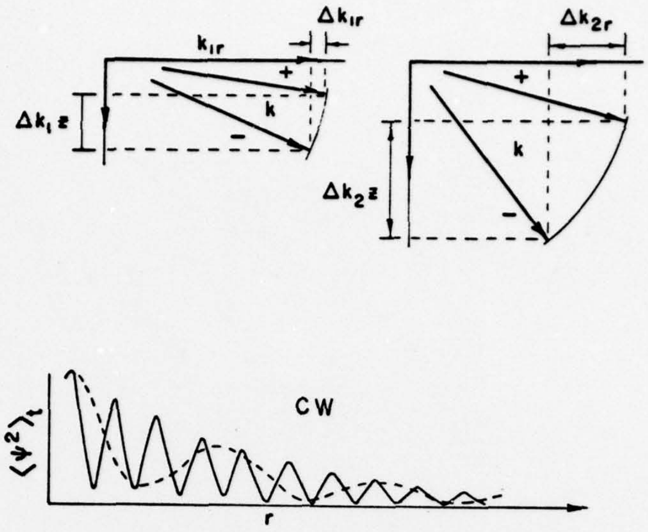

Figure 5. Effect of change of layer thickness upon mode interferences. The solid curve is for $h = h_o - \frac{1}{2} h_o$. The dotted curve is for $h = h_o + \frac{1}{2} h_o$.

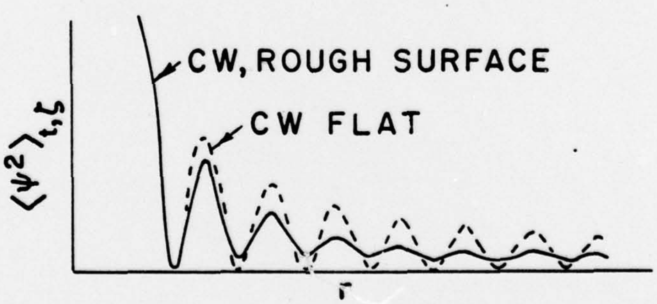

Figure 6. Acoustical field for smooth and rough surfaces. The average square of the acoustical field is given as a function of range for a smooth surface and slightly irregular surface. The wave-length of the surface irregularities is assumed to be large.

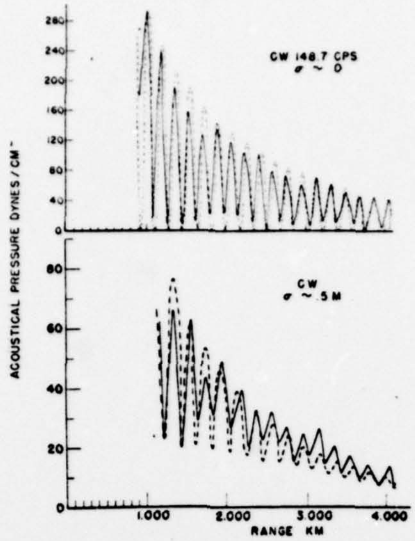

Figure 7. Comparison of theoretical and experimental data as a function of range and mean wave heights. The nearly flat, calm data ($\sigma \sim 0$) are shown on the upper part of the figure. A repeat of the same experiment was made with about sea state 2 and rms wave height about 0.5 m ($\sigma \sim 0.5$ m). The data and theory for this case are on the lower part of the figure. The data were taken in the Fire Island area in 21.6 m of water depth.